Period 1 Feb 5, 2025

3. a) Water flows from a storage tank at a rate of $2t$ liters per minute. Find the amount of water that flows out of the tank during the first 18 minutes.

$$\int_0^{18} 2T\,dT = T^2 + C \Big]_0^{18} \qquad 18^2 - 0^2 = 324$$

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b) For one hour, water flows into the storage tank at a rate of $5t$, in liters per minute. Find the amount of water that flows into the tank during the first 18 minutes.

$$\int_0^{18} 5T\,dT = \frac{5}{2}T^2 + C \Big]_0^{18} \quad \frac{5}{2}(18)^2 - 0^2$$

810

in

c) What does $W(0) = 1000$ represent?

amount in Tank when Time = 0 initial amount

d) Write an expression, with integral(s), for the total amount of water, $W(t)$. Use your expression to find the total amount of water in the tank after 18 minutes. Include units in your answer.

$$= 1000 + \int_0^{18} 5T\,dT - \int_0^{18} 2T\,dT = 1000 + 810 - 324$$

4. An object in rectilinear motion is moving along a horizontal line with velocity $v(t) = 3t^2 - 6t$ (in meters per second).

a) Find the total distance the object moved from $t = 1$ to $t = 4$.
b) Find the total displacement of the object from $t = 1$ to $t = 4$.
c) Find the average velocity of the object from $t = 1$ to $t = 4$.
d) For what times is the object at rest? Justify your answer. $T=0$ or $T=2$
e) At what time (if any) does the particle change directions. Justify your answer.

$T=0$ and $T=2$

$S(T) = \int v(T)\,dT$ distance $= \int |v(T)|\,dT$

object at Rest $v(T)=0$

object changes direction when $V(T)$ changes sign

First Find when $V(T)=0$

Then Test each side

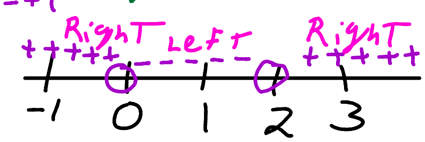
$V(T) = 3T^2 - 6T = 3T(T-2) = 0$ ← at Rest

$T=2$ or $T=0$

$V(-1) = 3(-1)^2 - 6(-1) = 3 + 6 = +9$

$V(1) = 3(1)^2 - 6(1) = 3 - 6 = -3$

$V(3) = 3(3)^2 - 6(3) = 27 - 18 = 9$



distance $\int_1^4 |v(T)|\,dT$

$\int_1^2 (3T^2 - 6T)\,dT$ $\int_2^4 (3T^2 - 6T)\,dT$

$T^3 - 3T^2 + c \Big[_1^2$ | $T^3 - 3T^2 + c \Big[_2^4$

$2^3 - 3(2)^2 - [1^3 - 3(1)^2]$ | $4^3 - 3(4)^2 - [2^3 - 3(2)^2]$

$8 - 12 - 1 + 3 = 11 - 13$ | $64 - 48 - 8 + 12 = 76 - 56 = 20$

-2

distance = |−2| + |20| = 22

displacement = −2 + 20 = 18

average velocity · width = displacement (1−4)

$$\frac{A \cdot \cancel{3}}{\cancel{3}} = \frac{18}{3}$$

Average velocity = 6 m/s

$$\int_{3}^{3x^2+2x} (5T-2)\,dT = \left(\left(5(3x^2+2x)-2\right)(6x+2)\right.$$

$$\frac{5}{2}T^2 - 2T + C \Big[_{3}^{3x^2+2x}$$

$$\frac{5}{2}(3x^2+2x)^2 - 2(3x^2+2x) - \underbrace{\left[\frac{5}{2}(3)^2 - 2(3)\right]}_{\text{constant}}$$

$$\frac{d}{dx}\left[\frac{5}{2}(3x^2+2x)^2 - 2(3x^2+2x)\right]$$

Example 7:

$$\frac{d}{dx}\left[\int_{x^2}^{3x} f(t)\,dt\right] = [F(3x)]3 - [F(x^2)](2x)$$

8)

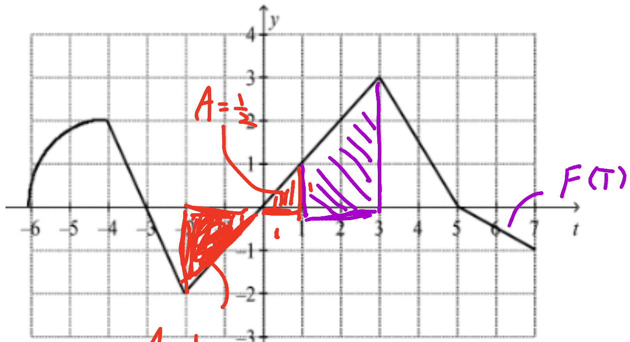
$$\frac{d}{dx}\left[\int_{2x}^{x^2} \frac{1}{2+e^t}\,dt\right] = \frac{1}{2+e^{x^2}} \cdot 2x - \frac{1}{2+e^{2x}} \cdot 2$$

$$\frac{2x}{2+e^{x^2}} - \frac{2}{2+e^{2x}}$$

9) $g(x) = \int_{6x}^{4x^2} \sqrt{1+t^4}\,dt$, What is $g'(x)$

$$\sqrt{1+(4x^2)^4}\,(8x) - \sqrt{1+(6x)^4}\,(6)$$

Let $g(x) = \int_1^x f(t)\,dt$, where f is the continuous function defined $[-6, 7]$ whose graph is shown below. Find the indicated values.



$A = \frac{1}{2} \cdot 2 \cdot 2 = 2$

10) $g(1)$

$g(1) = \int_1^1 F(T)\,dT = 0$

11) $g(3) = \int_1^3 F(T)\,dT$

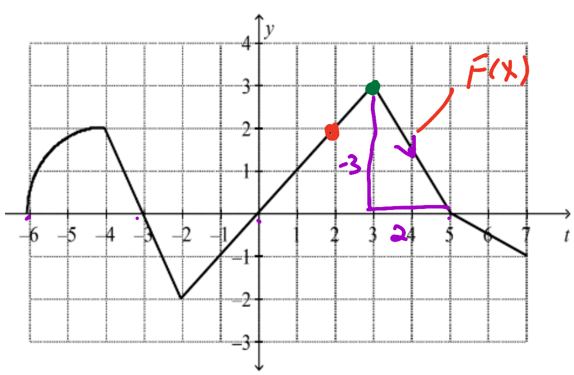
$\frac{1}{2}(1+3) \cdot 2 = 4$

12) $g(-2)$

$\int_1^{-2} F(T)\,dT = -\frac{1}{2} + 2$

$1\frac{1}{2}$

Let $g(x) = \int_1^x f(t)\,dt$, where f is the continuous function defined $[-6, 7]$ whose graph is shown below. Find the indicated values.



13) $g'(2)$

$g(x) = \int_1^x F(T)\,dT$

$\frac{d}{dx}[g(x)] = \frac{d}{dx}\left[\int_1^x F(T)\,dT\right]$

$g'(x) = F(x)$

$g'(2) = F(2) = 2$

14) $g'(3)$

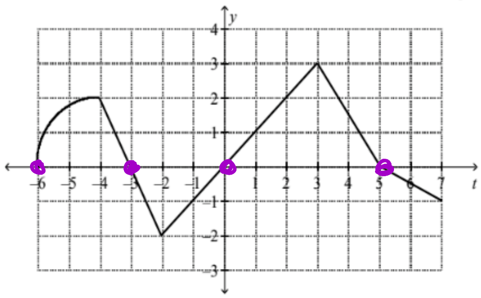
$g'(3) = F(3) = 3$

15) $g''(4)$

$g''(4) = F'(4) = \text{Slope of } F(x) \text{ at } x = 4$

$= \frac{-3}{2}$

17) Find the x-coordinate of each point where g has a horizontal tangent. For each of these points, determine whether g has a relative minimum, relative maximum or neither. Justify your answer.



$g'(x) = F(x)$

$g'(x) = 0$

$F(x) = 0$

$x = -6, -3, 0, 5$

$g''(x) = F'(x) =$ Slope of $F(x)$

Concave DN ∩ (Max) $= g''(x) = - = F'(x)$

Concave up ∪ (Min) $= g''(x) = + = F'(x)$

$g''(-3) = -$ = concave Down = Max $F(x)$ goes From + To −

$g''(0) = +$ = concave up = min $F(x)$ goes From − To +

$g''(5) = -$ = concave DN = Max $F(x)$ goes From + To −

18) For $-6 < x < 7$, find all x for which the graph of g is concave up.

$g''(x) = +$ concave up

$g(x) = \int_1^x F(T)\,dT$

$g'(x) = F(x)$

$g''(x) = F'(x) = $ Slope of $F(x)$

Concave up

$(-6,-4) \cup (-2,3)$

Slope $= +$

Slope $= +1$

Concave Down

$(-4,-2) \cup (3,5) \cup (5,7)$

Example 19: The population of the United States is growing at the rate of $P'(t) = 2.867(1.009)^t$ million people per year, where t is the number of years since 2015. If the population of the United States was 320 million people in 2015, what is the projected population in 2020.

Write an expression for the population of the US t years after 2015.

$320 + \int_0^T 2.867(1.009)^T \, dT$

Population in 2020

$320 + \int_0^5 2.867(1.009)^T \, dT$

$\int_0^5 2.867(1.009)^x dx$

$= 14.6609437935$

$320 + 14.6609$

334.66

Example 21: If the velocity particle is given by $v(t) = 2t$, meters/second and its initial position is 5 meters from the origin, write an expression for the position of the particle at any time t.

$5 + \int_0^T 2T \, dT = 5 + T^2 \Big|_0^T$

Expression for Amount Given Rates

Student Example: The rate, in bees per minute, at which bees leave the hive is given by $h(t) = t$ and the rate at which they enter is given by $b(t) = t^2$. If at $t = 3$ min, there are 10 bees in the hive, write an expression for the amount of bees at any given moment of time t. Use your expression to calculate the number of bees at $t = 6$.

Leave $h(T) = T$

enter $b(T) = T^2$

$10 = B_0 - \frac{9}{2} + 9 \Rightarrow 1 = B_0 - \frac{9}{2}$

$-9 \qquad -9 \qquad +\frac{9}{2} \qquad +\frac{9}{2}$

$B_0 = \frac{11}{2} = 5\frac{1}{2}$

in first 3 min

$\int_0^3 T \, dT$

$\frac{1}{2}T^2 \Big|_0^3 = \frac{9}{2} - 0 = \frac{9}{2}$

enter

$\int_0^3 T^2 \, dT = \frac{1}{3}T^3 \Big|_0^3$

$\frac{27}{3} - \frac{0}{3} = 9$

bee's $= 5\frac{1}{2} - \int_0^T T \, dT + \int_0^T T^2 \, dT$

bee's at $T = 6 \Rightarrow 5\frac{1}{2} - \int_0^6 T \, dT + \int_0^6 T^2 \, dT = 10 - \int_3^6 T \, dT + \int_3^6 T^2 \, dT$

Example 5

A pizza with a temperature of 95°C is put into a 25°C room when $t = 0$. The pizza's temperature is decreasing at a rate of $r(t) = 6e^{-0.1t}$ °C per minute. Estimate the pizza's temperature when $t = 5$ minutes.

Solution

How much pizza has cooled

Temperature $= 95 - \int_0^5 6e^{-0.1t}\,dt = 71.392$°C

$$\int_0^5 6e^{-0.1x}dx$$

95-23.60816 = 71.392

$= 23.6081604172$